Lecture 9: Load Balancing & Resource Allocation

Introduction

- Moler's law, Sullivan's theorem give upper bounds on the speed-up that can be achieved using multiple processors.
- But to get these need to "efficiently" assign the different concurrent processes that make up a concurrent program on the available processors.
- This is called *Load Balancing*.
- Load balancing is a special case of more general *Resource Allocation Problem* in a parallel/distributed system.
- In the load balancing situation, resources are processors.
- Before clarifying load balancing problem need to formalise models of the concurrent program and concurrent system.
- To do this, we can use methods such as *Graph Theory*.

Sources of Parallel Imbalance

- Individual processor performance
  - Typically in the memory system
- Too much parallelism overhead
  - Thread creation, synchronization, communication
- Load imbalance
  - Different amounts of work across processors (comp: comms ratio)
  - Processor heterogeneity (maybe caused by load distribution)
- Recognizing load imbalance
  - Time spent at synchronization is high/uneven across processors

Aside: Graph Theory

- Directed graph are useful in the context of load balancing
- Nodes can represent tasks and the links representing data or communication dependencies
- Need to partition graph so that to minimize execution time.
- The graph partition problem is formally defined on data represented in the form of a graph

$$G = (V, E) \text{ with } V \text{ vertices and } E \text{ edges}$$

- It is possible to partition G into smaller components with specific properties.
- For instance, a k-way partition divides the vertex set into k smaller components.
- A good partition is defined as one in which the number of edges running between separated components is small.

Graph Theory (cont'd)

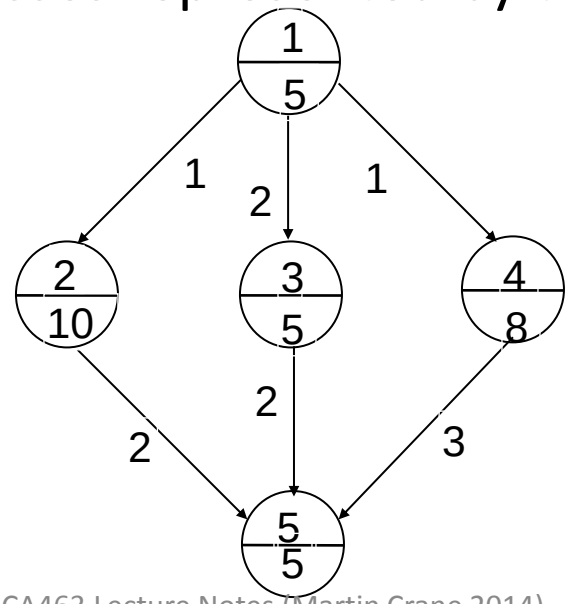
- Partition G such that
  - $\{V\} = \{V_1\} \cup \{V_2\} \cup \cdots \cup \{V_n\}$ with $|V_i| \approx |V|/n$
  - As few of $\{E\}$ connecting $|V_i|$ with $|V_j|$ as possible

- If $\{V\}$ = {tasks}, each unit cost, edge $e=(i,j)$ (comms between task i and task j), and partitioning means
  - $\{V\} = \{V_1\} \cup \{V_2\} \cup \cdots \cup \{V_n\}$ with $|V_i| \approx |V|/n$ i.e. load balancing
  - Minimize $\{E\}$ i.e. minimize comms

- As optimal graph partitioning is NP complete, so use heuristics
- Trades off between partitioner speed & with quality of partition
- Better load balance costs more and law of diminishing returns?

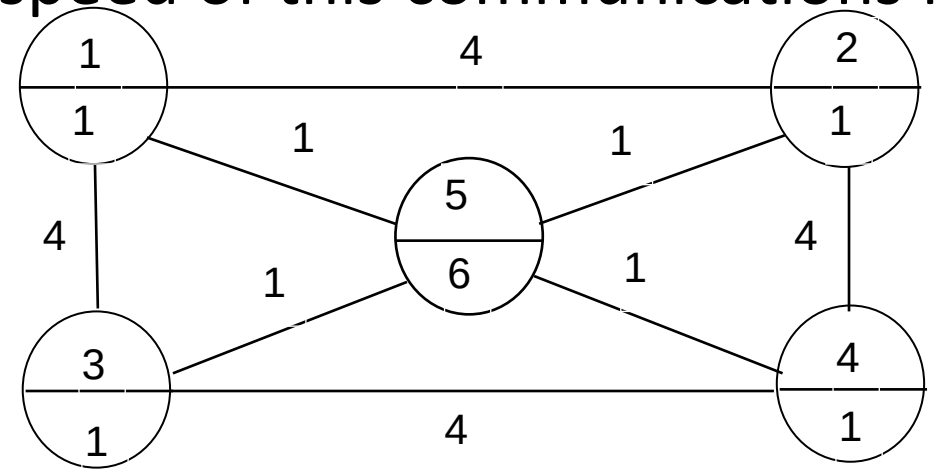
Formal Models in Load Balancing: Task Graphs

- A task graph is a directed acyclic graph where
  - nodes denote the concurrent processes in a concurrent program
  - edges between nodes represent process comms/synchronisation
  - nodal *weight* is the computational load of the process the node represents
  - edge *weight* between two nodes is the amount of comms between two processes represented by the two nodes.

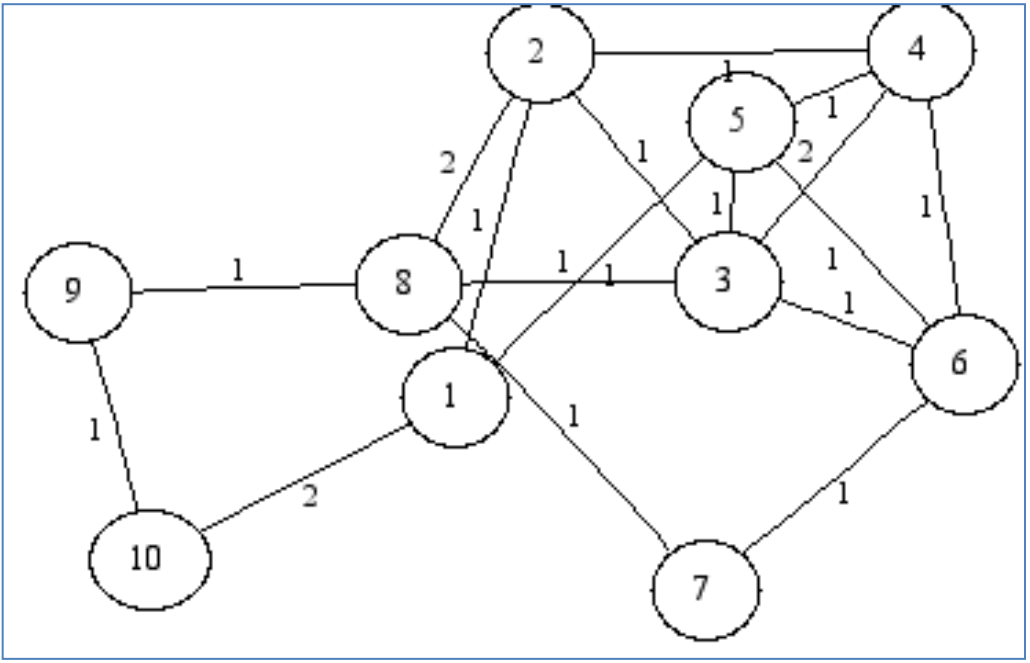


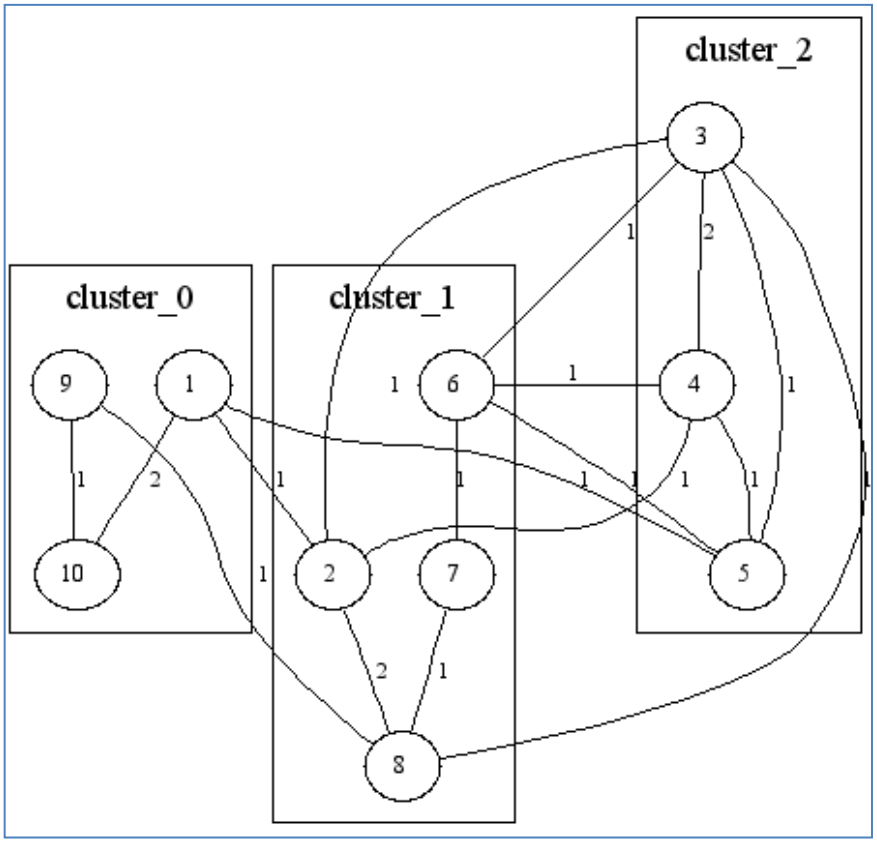
Formal Models in Load Balancing: Processor Graphs

- The processor graph defines the configuration of the parallel or distributed system.
- Each node represents a processor & the nodal weight is the computation speed of this processor.
- The edges between nodes represent the communication links between the processors represented by the nodes.
- Edge weight is the speed of this communications link.



Load Balancing Based on Graph Partitioning: Typical Example





- The Nodes represent tasks
- The Edges represent communication cost
- The Node values represent processing cost
- A second node value could represent reassignment cost

Load Balancing: The Problem

- To partition a set of interacting tasks among a set of interconnected processors to maximise "performance".
- Basically the idea in load balancing is to balance the processor load so they all can proceed at the same rate.
- However formally can define maximising "performance" as:
  - minimising the *makespan*[1], C_{max} :

$$\min(C_{max}) = \min(\max_{1 \le i \le n} C_i)$$

  - minimising the response time, the total idle time, or
  - any other reasonable goal.
- A general assumption that is made is that the comms between tasks on the same processor is much faster than that between two tasks on different processors.
- So intra-processor comms is deemed to be instantaneous.

[1] where *makespan* is defined as the maximum completion time of any of the n tasks

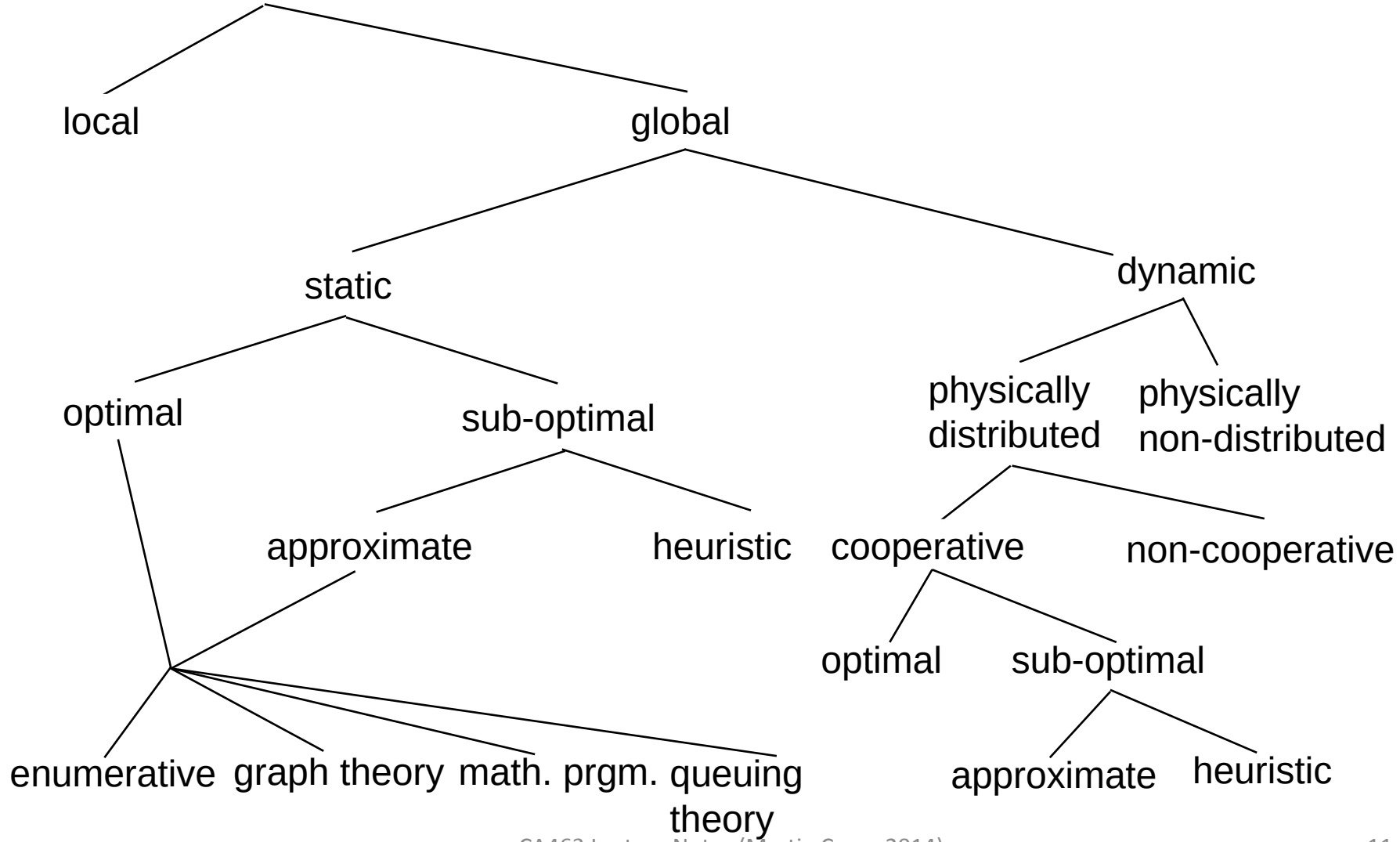
Load Balancing: Allocation & Scheduling

- Load Balancing has two aspects:
  - the *allocation* of the tasks to processors, and
  - the *scheduling* of the tasks allocated to a processor.
- Allocation is usually seen as the more important issue.
  - As a result some load balancing algorithms only address allocation.
- Complexity of the problem:
  - Find an allocation of n arbitrarily intercommunicating tasks,
  - constrained by precedence relationships,
  - to an arbitrarily interconnected network of m processing nodes,
  - meeting a given deadline

    this is an NP complete problem.
- Finding $\min(C_{max})$ for a set of tasks, where any task can execute on any node and is allowed to pre-empt another task, is NP complete even when the number of processing nodes is limited to two.

Casavant & Kuhl's Taxonomy

- A hierarchical taxonomy of algorithms is by Casavant and Kuhl.



Casavant & Kuhl (cont'd): Static V Dynamic

- Static Algorithms:
  - nodal assignment (once made to processors) is fixed
  - use only info about the average behaviour of the system.
  - ignore current state/load of the nodes in the system.
  - are obviously much simpler.
- Dynamic Algorithms:
  - use runtime state info to make decisions
  - i.e. can tasks be moved from one processor as system state changes?
  - collect state information and react to system state if it changed
  - are able to give significantly better performance

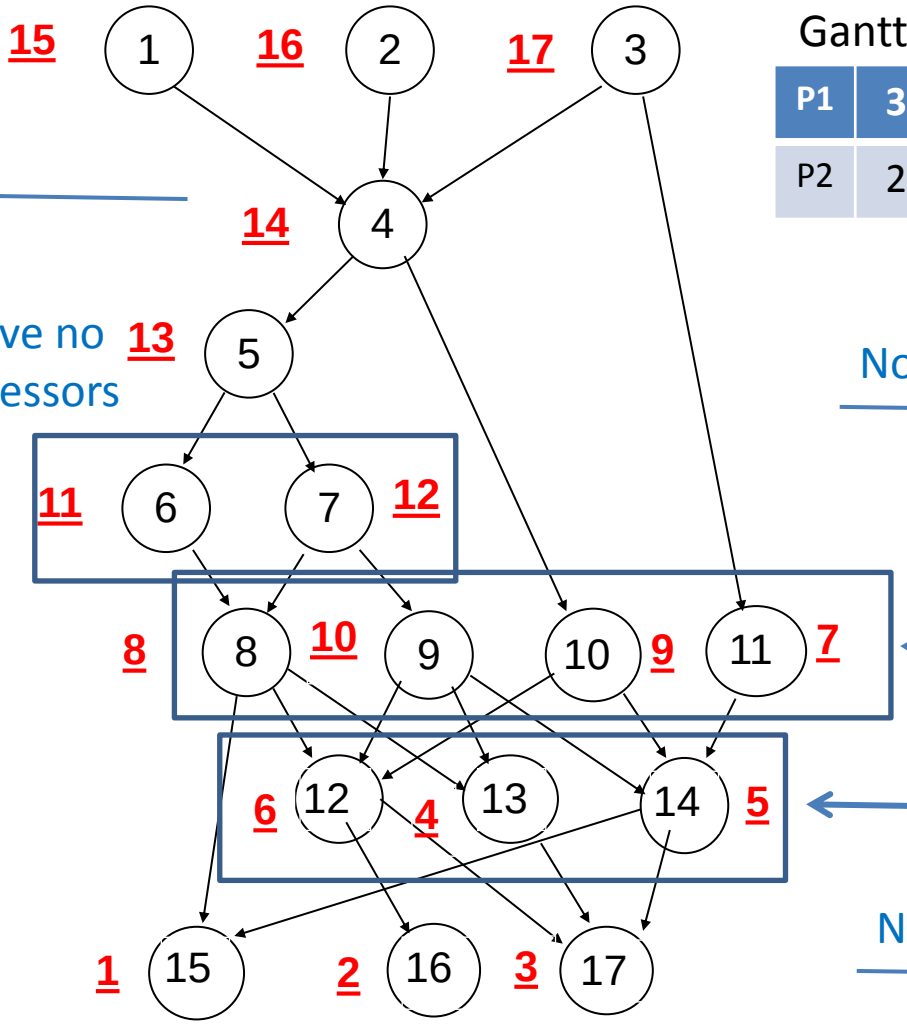
Casavant & Kuhl (cont'd): Centralized V Distributed

- Centralized Algorithms:
  - collect info to server node and it makes assignment decision
  - can make efficient decisions, have lower fault-tolerance
  - must take account of info collection/allocation times
- Distributed Algorithms:
  - contains entities to make decisions on a predefined set of nodes
  - avoid the bottleneck of collecting state info and can react faster
  - don't have to take account of info times

Load Balancing: Coffman's Algorithm

- This is an optimal static algorithm that works on arbitrary task (program) graphs.
- Since generally, the problem is NP-complete, some simplifying assumptions must be made:
  1. All tasks have the same execution time.
  2. Comms negligible versus computation. Precedence ordering remains.
- The Algorithm
  1. Assign labels $1, \ldots, t$ to the t terminal (i.e. end) tasks.
     a) Let labels $1, \ldots, j-1$ be assigned, and let S be the set of tasks with no unlabelled successors.
     b) For each node x in S define $l(x)$ as the decreasing sequence of the labels of the immediate successors of x.
     c) Label x as j if $l(x) \leq l(x')$(lexicographically) for all x' in S.
  2. Assign the highest labelled ready task to the next available time slot among the two processors.

Coffman's Algorithm: Example



Gantt Chart

P1	3	1	4	5	7	9	12	13	17
P2	2	11		10	6	8	14	16	15

Nodes	Inv Lex
6	8
7	10 8

These Nodes have no Unlabelled Successors

Nodes	Inv Lex
8	641
9	654
10	65
11	5

These Nodes have no Unlabelled Successors

These Nodes have no Unlabelled Successors

Nodes	Inv Lex Order of Successors
14	31
13	3
12	32

Scheduling Algorithms

- Concepts of *load balancing* & *scheduling* are closely related.
- The goal of scheduling is to maximize system performance, by switching tasks from busy to less busy/ idle processors
- A scheduling strategy involves two important decisions:
  1. determine tasks that can be executed in parallel, and
  2. determine where to execute the parallel tasks.
- A decision is normally taken either based on prior knowledge, or on information gathered during execution.

Scheduling Algorithms: Difficulties

- A scheduling strategy design depends on the tasks' properties:

a) Cost of tasks
   - do all tasks have the same computation cost?
   - if not, when are costs known? before execution, on creation, or on termination?

b) Dependencies between tasks
   - can we execute the tasks in any order?
   - if not, when are task dependencies known?
   - again, before execution, when the task is created, or only when it terminates?

c) Locality
   - is it important that some tasks execute in the same processor to reduce communication costs?
   - when do we know the communication requirements?

- Have come up against a lot of these ideas already in MPI Lectures

Scheduling Algorithms: Differences

- Like Allocation Algorithms, Scheduling Algorithms can be either *Static* or *Dynamic*.
- A key question is when certain information about the load balancing problem is known.
- Leads to a spectrum of solutions:

1. *Static scheduling*:

- In this all info is available to the job scheduling algorithm
- Then this is able to run before any real computation starts.
- For this case, we can run off-line algorithms, eg graph partitioning algorithms.

Scheduling: Semi-Static Algorithms

2. Semi-Static Scheduling:

- In this case, info about load balancing may be known
  - program startup, or
  - beginning of each timestep, or
  - at other well-defined points in the execution of the program.
- Offline algorithms may be used even though the problem has dynamic aspects. eg Kernighan-Lin Graph Partitioning Algorithm
- Kernighan-Lin (KL) is a $O(n^2 \log n)$ heuristic algorithm for solving the graph partitioning problem.
- It is commonly applied as a solution to the Travelling Salesman Problem (TSP) which, ordinarily, is NP complete.

Scheduling: Semi-Static Algorithms (cont'd)

- KL tries to split V into two disjoint subsets A, B of equal size.
- Partitioned such that sum T of the weights of the edges between nodes in A and B is minimized.
- Proceeds by finding an optimal set of interchanges between elements of A, B maximizing $T_{old} - T_{new}$ (iterating as necessary)
- It then executes the operations, partitioning V into A and B.
- Kernighan-Lin has many applications in such areas as diverse as:
  - Circuit Board Design (where edges represent solder on a circuit board and need to minimize crossings between components represented by vertices) and
  - DNA sequencing (where edges represent a similarity measure between DNA fragments and the vertices represent DNA fragments themselves).

Scheduling: Dynamic Algorithms

3. *Dynamic Scheduling:*

- Here load balancing info is only known mid-execution.
- This gives rise to sub-divisions under which dynamic algorithms can be classified:
  a. *source-initiative algorithms*, where the processor that generates the task decides which processor will serve the task, and
  b. *server-initiative algorithms*, where each processor determines which tasks it will serve.
- Examples of source-initiative algorithms are *random splitting*, *cyclical splitting*, and *join shortest queue*.
- Examples of server-initiative algorithms are *random service*, *cyclical servicing*, *serve longest queue* and *shortest job first*.

Scheduling: Dynamic Algorithms (cont'd)

- Server-initiative algorithms tend to out-perform source-initiative algorithms, with the same information content if the communications costs are not a dominating effect.
- However, they are more sensitive to distribution of load generation, and deteriorate quickly when one load source generates more tasks than another.
- But in heavily loaded environments server-initiative algorithms dominate source-initiative algorithms.

Scheduling in Real Time Systems (RTS)

- The goal of scheduling here is to guarantee:
  - that all critical task meet their deadlines and
  - that *as many as possible* essential tasks meet theirs.
- RTS Scheduling can be *synchronous* or *asynchronous*.

1. *Synchronous Scheduling Algorithms*

- These are static algorithms in which the available processing time is divided by hardware clock into *frames*.
- Into each frame a set of tasks are allocated which will be guaranteed to be completed by the end of the frame.
- If a task is too big for a frame it is artificially divided into highly dependent tasks such that the smaller tasks can be scheduled into the frames.

RTS Scheduling (cont'd)

2. *Asynchronous Scheduling*

- This can be either *static* or *dynamic*.
- In general *dynamic* scheduling algorithms are preferred as static algorithms cannot react to changes in state such as h/w or s/w failure in some subsystem.
- Dynamic Asynchronous Scheduling Algorithms in a hard real time system must still guarantee that all critical tasks meet their deadlines under specified failure conditions.
- So critical tasks are scheduled statically and replicates of them are statically allocated to several processors and that the active state information of the task is also duplicated.
- In the event of a processor failure the state information is sent to a duplicate of the task and all further inputs are rerouted to the replicate task.